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Numeric Modelling

Simple Modeling of Laplace's Equation on Potential Using Finite Difference

Delia Meldra[a,], Yopy Mardiansyah[b]*

[a] *Industrial Engineering Study Program, Ibnu Sina University, Batam City, Riau Islands 29444, Indonesia*
[b] *Industrial Engineering Study Program, Batam Institute of Technology, Batam, Riau Islands, 29425, Indonesia*

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CORRESPONDING AUTHOR

phone:

E-mail: dmeldra@gmail.com

A B S T R A C T

Geo-electricity is a method used to measure potential on the earth's surface. The second-order Laplace equation was used in natural potential measurements. The finite difference method was applied to solve this Laplace equation in order to obtain a numerical solution. The solution depicted the distribution of source potential derived from various sides of the modeling domain. This research discussed potential distribution modeling using the finite difference method to solve the Laplace equation. The Laplace equation is a second-order partial differential equation that describes potential distribution in a medium without electrical charge. The finite difference method was used to discretize the problem domain into grid points, enabling a numerical solution to the equation. This research developed a mathematical model based on Taylor series expansion and implemented it in MATLAB code. Two modeling scenarios were presented: (1) potential sources on the right and left sides of the domain, which resulted in a semi-circular distribution pattern, and (2) potential sources on all four sides of the domain, which resulted in a convergent distribution pattern towards the center. The modeling results demonstrated that the finite difference method was effective in visualizing potential distribution for various source configurations. The study concluded that increasing the number of discrete points in the finite difference method could enhance the detail and accuracy of the model.

1. INTODUCTION

Geoelectric method is a method that calculates the potential on the earth's surface, one of which is self-potential. Self-potential is the measurement of natural potential differences on the earth's surface. The Self-Potential method was first initiated by Robert Fox in 1830 [1]. Potential difference is analytically described in terms of the Laplace equation. The Laplace equation is a second-order equation that can be solved numerically and modeled. There are two types of modeling: forward and inverse modeling. Forward modeling is modeling done to obtain the response of the model made based on known physical properties.

Analysis of the Self-Potential method can be done by modeling. There are two types of modeling: forward and inverse modeling. Forward modeling is modeling done to obtain the response of the model made based on known physical properties [2].

In this study, the Laplace equation is solved using the finite difference method so that the potential difference distribution can be modeled. Finite difference is one of the numerical techniques used to solve the Laplace differential equationl [3]. In finite difference the domain is divided into several discrete points. The Laplace equation is solved by finite difference using Taylor expansion [4]. The number of discrete points will affect the accuracy and detail of the resulting model.

2. LITERATURE REVIEW

2.1. Potential

This method is called Self-Potential because the potential is generated by a number of natural sources from the earth without any injection into the subsurface. The value of the potential difference measured at the surface is less than a millivolt to one volt, and the sign (negative or positive) of the potential is an important factor for the interpretation of Self-Potential anomalies. This method is usually used for shallow exploration, around 100 m, but in the modeling domain it can be up to 1.5 km. Self-Potential anomalies can be generated by diffusion and membrane mechanisms, bioelectric potential, streaming potential, and mineral potential [5].

Diffusion and membrane potential are related to the concentration gradient and the movement of ions below the surface, resulting in an electric potential difference. Water pumping and ion filtration by plant roots will generate bioelectric potential. Streaming potential (electrokinetic potential) is related to the flow of groundwater and other fluids containing electrolytes through faults [6].

2.2. *Geoelectric*

Geoelectric method is one of the geophysical methods that studies the nature of electricity flow in the earth and how to detect it in the earth and how to detect it on the earth's surface [7]. This includes the measurement of potential, current and electromagnetic fields that occur either naturally or due to the injection of current into the earth. To detect the subsurface structure of the earth usually utilizes the potential distribution due to a single current source on the surface of the homogeneous earth which can be shown by Figure 1.

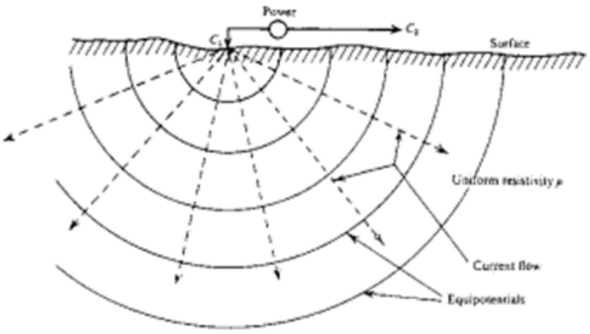

Figure 1. Current and voltage distribution with a single current source [5]

2.3. Finite Difference

Finite difference is a method that uses derivatives of a formula to solve a problem [8]. Derivatives are calculated using very small but finite values. As it is known that every first-order differential formula contains an integral constant [9]. Therefore, the calculation requires a boundary condition that will fulfill the value of the integral constant.

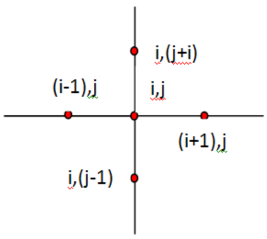

Figure 2. Titik-Titik Diskrit Finite difference pada Koordinat 2-Dimensi

The finite difference solution results are shown only at discrete points, while the general variation of the calculation results of each point cannot be known by the finite difference method but can be seen by interpolation.

2.4. MATLAB

MATLAB is a computer program that can help solve various mathematical problems that we often encounter in the technical field [10]. We can use MATLAB to quickly find solutions to various numerical problems, from the most basic, such as a system of 2 equations with 2 variables, such as , to complex ones, such as second-order equations, polynomial roots, data set interpolation, matrix calculations, and numerical methods. And one of the very important and useful things about MATLAB is the ability to draw various types of graphs, so that it can visualize complex data and functions in 1-dimensional, 2-dimensional, or 3-dimensional form.

3. METHODOLOGY

3.1. Data Collection Technique

This research used an observational and literature approach to help collect research information. The observation aimed to obtain direct information from the testing of the research model, while literature was used to gather information from modeling. The stages undertaken in the research, concept development, or case resolution were written in the methodology section.

3.2. Laplace Equation

For a space or medium without electric charge, $\nabla^2 V=0$, we can write the Laplace equation,

$$\nabla^2 V=0 \tag{1}$$

Operator ∇^2 is called the Laplacian of V. The three-dimensional Laplace equation for :

(a) cartesian coordinate system is

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{2}$$

(b) cylindrical coordinate system is

$$\nabla^2 V = \frac{\partial}{\rho \partial \rho}\left(\frac{\rho \partial V}{\partial \rho}\right) + \frac{\partial^2 V}{\rho^2 \partial \phi^2} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{3}$$

(c) the spherical coordinate system is

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^s \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} \tag{4}$$

The solution of the one-dimensional Laplace equation is developed to obtain the relationship between charge Q and potential difference V so that the capacity of a capacitor can be determined. An example of using the one-dimensional Laplace equation to determine the capacitance formula.

4. RESULTS AND DISCUSSION

4.1. Laplace Equation Using Finite Difference

The Laplace equation for potential used the Finite Difference method:

$$\nabla^2 V = 0 \tag{5}$$

$$\nabla^2 V_{(x,y)} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \tag{6}$$

Using the finite difference approach for the potential function $V_{(r)}$ [11] Obtained through partial differential equations in the equation (5). In the figure there are points at i+1, i, and i-1 so that the value of the function $V_{(x,y)}$ present in V_{i+1}, V_i, V_{i-1}. By using Taylor series expansion, then:

$$V_{i-1} = V_i - \frac{\partial V}{\partial x}\Big|_i h + \frac{\partial^2 V}{\partial x^2}\Big|_i \frac{h^2}{2!} - \ldots + \frac{\partial^n V}{\partial x^n}\Big|_i \frac{h^n}{n!} \quad (7)$$

with n = 0, 2, 4, … positive sign (+)

n = 1, 3, 4, … negative sign (-)

$$V_{i+1} = V_i + \frac{\partial V}{\partial x}\Big|_i h + \frac{\partial^2 V}{\partial x^2}\Big|_i \frac{h^2}{2!} + \ldots + \frac{\partial^n V}{\partial x^n}\Big|_i \frac{h^n}{n!} \quad (8)$$

with n = 0, 1, 2, 3, … positive sign (+)

$\Big|_i$ = calculation at point i

Summation in equation (6) and (7) :

$$V_{i-1} + V_{i+1} = 2V_i + \frac{\partial^2 V}{\partial x^2}\Big|_i h^2 + \frac{\partial^4 V}{\partial x^4}\Big|_i \frac{h^4}{12} \quad (9)$$

so,

$$\frac{\partial^2 V}{\partial x^2}\Big|_i = \frac{V_{i+1} - 2V_i + V_{i-1}}{h^2} \quad (10)$$

The right side of the equation is a second-order finite difference approximation that is accurate. Then, by eliminating the equation (9) and (10)

$$V_{i+1} - V_{i-1} = \frac{\partial V}{\partial x}\Big|_i 2h + \frac{\partial^3 V}{\partial x^3}\Big|_i \frac{h^3}{3} \quad (11)$$

sehingga,

$$\frac{\partial V}{\partial x}\Big|_i = \frac{V_{i+1} - V_{i-1}}{2h} \quad (12)$$

With the same treatment for the y-axis, then:

$$\frac{\partial^2 V}{\partial y^2}\Big|_j = \frac{V_{j+1} - 2V_j + V_{j-1}}{h^2} \quad (13)$$

$$\frac{\partial V}{\partial y}\Big|_j = \frac{V_{j+1} - V_{j-1}}{2h} \quad (14)$$

from equation (12) and (13),

$$\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)\Bigg|_{i,j} = \frac{V_{i+1} - 2V_i + V_{i-1}}{h^2} + \frac{V_{j+1} - 2V_j + V_{j-1}}{h^2} \quad (15)$$

The Laplace Equation after applying finite difference becomes,

$$\nabla^2 V = 0 \quad (16)$$

$$\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2}\right)\Bigg|_{i,j} = 0 \quad (17)$$

$$V_{i,j} = \frac{1}{4}\left(V_{i+1,j} + V_{i-1,j} + 2V_{i,j+1} + 2V_{i,j-1}\right) \quad (18)$$

4.2. Forward Modeling of the Laplace Equation

Equation (18), which has been obtained analytically, will be modeled using a MATLAB simulation,

```
% ====================================================================
% Solusi Persamaan Laplace
% Kasus Potensial
% ====================================================================
% Delia Meldra

clear all;
close all; clc

x=0:1:200;
y=0:1:200;

V(201,201)=0;

%% Boundary Condition
for i=1:201
    V(i,1)=20;
end
for i=1:201
    V(1,i)=0;
end
end
for i=1:201
    V(201,i)=0;
end

n=2000;
for i=1:n;
    for ix=2:200;
            V(iy,ix)=(V(iy-1,ix)+V(iy+1,ix)+V(iy,ix-1)+V(iy,ix+1))/4;
        end
    end
end

[X,Y]=meshgrid(x,y);
contour(X,Y,V)
```

Figure 3. MATLAB Coding for the Laplace Equation

4.3. Modeling of distribution

After Equation (18) was completed in the coding using the finite difference method, it can be seen in Figures 4 and 5,

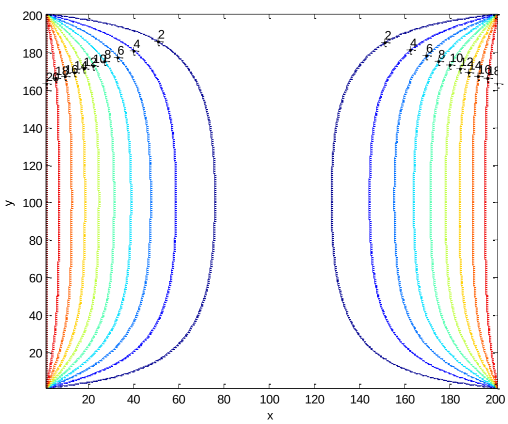

Figure 4. Distribution Model with Right and Left Source Inputs

Figure 4 displayed the Laplace equation with sources on the right and left, with sources located along the right and left sides, resulting in a potential distribution that formed a semicircle. Using the finite difference method clarified the shape of the distribution.

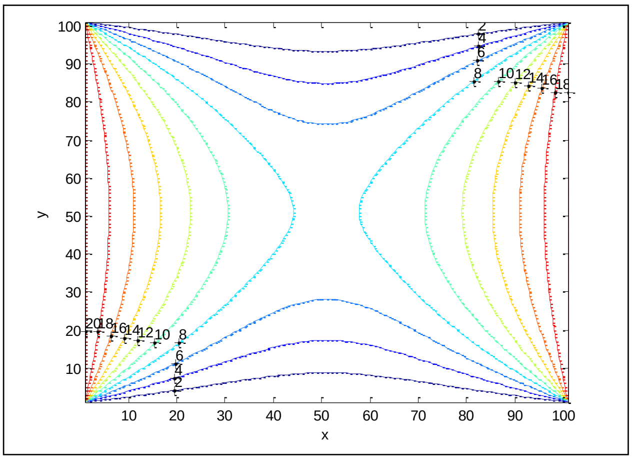

Figure 5. Distribution Model with Source Inputs from Right, Left, Top, and Bottom

Figure 5 shows the Laplace equation with sources from each side: right, left, top, and bottom. The sources are located along these sides, causing the potential distribution to converge toward the center.

5. CONCLUSION

The solution of the Laplace equation using the finite difference method facilitates the simulation or modeling of potential distribution. The development of this fundamental knowledge can be applied to various problems. By increasing the number of points in the finite difference method, more detailed results can be obtained.

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